**Applied (Word) Problems NoteSheet**

**Distance – Rate – Time**

**Uniform Motion**

Focus: To solve word problems with uniform motion by using the formula $d=rt$.

**Rate – Time – Distance**

- **Rate** is how fast an object is traveling.
- **Time** is how long an object has been traveling.
- **Distance** is how far an object has traveled.

Here are three helpful hints that will aid you when you get ready to set up your equations:

There are three types of DRT (uniform motion) problems:

1) if the objects are traveling in opposite directions and their times are the same: **ADD THE DISTANCES TOGETHER**;

2) if the objects are traveling in the same direction and you want to know when one will catch up or overtake the other: **SET THE DISTANCES EQUAL TO EACH OTHER**;

3) if the objects are traveling in the same direction with one going faster than the other and you want to know when they will be a certain distance apart: **SUBTRACT THE DISTANCES**

A typical problem involving distance and the formula $d = rt$ is usually entitled a **uniform motion** problem. The problem will have something to do with objects moving at a constant rate of speed or an average rate of speed.

**The formula Distance = Rate x Time expresses one of the most frequently used relations in algebra.**

Since an equation remains true as long as you divide through by the same non-zero element on each side, this formula can be written in different ways:

- To find **rate**, divide through on both sides by **time**:
  \[
  \frac{\text{Distance}}{\text{Time}} = \text{Rate}
  \]

  *Rate* is distance (given in units such as miles, feet, kilometers, meters, etc.) divided by time (hours, minutes, seconds, etc.). Rate can always be written as a fraction that has distance units in the numerator and time units in the denominator, e.g., 25 miles/1 hour.

- To find **time**, divide through on both sides by **rate**:
  \[
  \frac{\text{Distance}}{\text{Rate}} = \text{Time}
  \]

When using this equation, it's important to keep the units straight. For instance, if the rate the problem gives is in miles per hour (mph), then the time needs to be in hours, and the distance in
**Distance - Rate - Time**

If the time is given in minutes, you will need to divide by 60 to convert it to hours before you can use the equation to find the distance in miles. Always make your units match: if the time is given in fortnights and the distance in furlongs, then the rate should be given in furlongs per fortnight.

You can see why this is true if you look carefully at how the units are expressed. Say a car is travelling at 30 mph and you want to figure out how far it will go in 2 hours. You can use the formula:

\[
\text{Rate} \times \text{Time} = \text{Distance}
\]

\[
30 \text{ miles/hr} \times 2 \text{ hours} = 60 \text{ miles}
\]

The hours cancel, leaving only miles.

What if you want to calculate the number of miles a car travelling 30 mph goes in 120 minutes?

Since 120 minutes is equal to two hours (60 minutes in one hour x 2 hours = 120 minutes), we should get the same distance of 60 miles, but we will not get the answer this way:

\[
\frac{30 \text{ miles}}{\text{hour}} \times 120 \text{ minutes} = 3600 \text{ mile-minutes}
\]

Now, 3600 mile minutes per hour isn't very helpful, since we'd like our answer in miles. We need to divide by 60 minutes per hour:

\[
\frac{3600 \text{ mile-minutes}}{\text{hour}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = 60 \text{ miles}
\]

Although we can find an answer this way in the correct units, a better method would be to convert minutes to hours before using the formula.

Remembering to be careful about units, let's look at a problem.

Superheroes Liza and Tamar leave the same camp and run in opposite directions. Liza runs 1 mile per second (mps) and Tamar runs 2 mps. How far apart are they in miles after 1 hour?

To begin, we can either convert rates to miles per hour, or we can convert the time to seconds.

Let's convert from miles per second to miles per hour.

\[
\frac{3600 \text{ miles}}{\text{hour}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = 60 \text{ miles}
\]

The hours and the minutes cancel, leaving only miles.

Although we can find an answer this way in the correct units, a better method would be to convert minutes to hours before using the formula.


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Back to the problem. How far does Liza run in one hour? We know her rate (3600 mph) and the time that she runs (one hour), so we can use the formula:

\[
\frac{\text{miles}}{3600} \times 1 \text{ hour} = 3600 \text{ miles} \frac{\text{hour}}{}
\]

This makes sense because, by definition, if Liza’s speed is 3600 miles per hour, then she runs 3600 miles in an hour.

Tamar, whose speed is 7200 miles per hour, will run 7200 miles in an hour.

How far apart will the two runners be after an hour? The answer is simply the sum of the distance each runs in an hour: \(3600 + 7200 = 10,800\) miles apart.

Since the earth has a circumference of about 24,000 miles at its equator, that’s a little less than halfway around the world!

**These problems, however, can be tricky:**

Karen can row a boat 10 kilometers per hour in still water. In a river where the current is 5 kilometers per hour, it takes her 4 hours longer to row a given distance upstream than to travel the same distance downstream. Find how long it takes her to row upstream, how long to row downstream, and how many kilometers she rows.

One of the best ways to start a problem like this is to make a table that uses all the information you have been given. Let’s make one for the information we have about the distance, rate, and time Karen travels when she is going both upstream and downstream. We’ll call the time it takes to row downstream \(x\), which means that the time it takes to row upstream is \(x + 4\).

We’ll start by calculating Karen’s rates going upstream and downstream. When she is traveling against the current, she won’t be able to row 10 kilometers/hour. Her speed relative to the shore will only be 5 kilometers per hour because the force of the current, which is flowing at 5 kilometers/hour, slows her rate by 5 km/hour. When Karen is rowing downstream, however, the current helps her go faster, so she moves \(10 + 5 = 15\) km/hour.

We can use the formula, written as \(\text{Rate} \times \text{Time} = \text{Distance}\):

<table>
<thead>
<tr>
<th></th>
<th>Rate (km/hr)</th>
<th>Time (hr)</th>
<th>Distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downstream</td>
<td>15</td>
<td>(x)</td>
<td>15(x)</td>
</tr>
<tr>
<td>Upstream</td>
<td>5</td>
<td>(x + 4)</td>
<td>5(x + 4)</td>
</tr>
</tbody>
</table>

Because Karen goes the same distance upstream and downstream, we know that the two expressions of distance - for upstream and downstream - must be equal; we can set the upstream distance equal to the downstream distance. This produces the following equation, which we solve for \(x\):

\[
\text{Statement of original equation: } 15x = 5(x + 4)
\]

\[
\text{Distributing on right side: } 15x = 5x + 20
\]
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Subtracting 5x from both sides: \(10x = 20\)

Dividing both sides by 10: \(x = 2\)

\(x\) equals the time it takes Karen to row downstream, or 2 hours. Since it takes her four hours longer to row upstream, this time will be \(2 + 4 = 6\) hours.

How many kilometers does she row? Look at the distance column in the table. Since \(x\) is in hours, Karen's downstream distance is \(15 \times 2 = 30\) kilometers.

The problem states that Karen rows the same distance upstream as down. Let's check our work... yes, \(5(2+4) = 5 \times 6 = 30\) kilometers.

As is frequently the case with word problems, setting up the equations is the hardest part. Once that's done, the rest is relatively easy. Remember always to answer what the question asks - don't stop once you've solve for \(x\), because that may be only part of what the question asked - and **always** check your answer.
1. If your speedometer is incorrect, and you travel one mile in 45 seconds, how fast are you driving?

2. Bernice is cycling around a track at 15 mph. Betty starts at the same time, but only goes 12 mph. How many minutes after they start will Bernice pass Betty if the track is 1/2 mile long? They are moving in the same direction.

3. You are traveling 55 mph over a bridge that is 4260 ft. long. How long does it take to cross the bridge?

4. A hot air balloon covered 2100 miles in 7 days. If it covered 50 miles more each day than the day before, how many miles did it cover each day?

5. Jack climbed up the beanstalk at a uniform rate. At 2 P.M. he was one-sixth the way up and at 4 P.M. he was three-fourths the way up. What fractional part of the entire beanstalk had he climbed by 3 P.M.? At what time did he start climbing? When will he get to the top? How long was his trip? And how tall was the beanstalk anyway?

6. A jet is flying 2400 miles from Hawaii to San Francisco. In still air, it flies at 600 mph. There is a 40 mph tailwind. How many hours after takeoff would it be faster to just go to San Francisco than to return to Hawaii in the case of an emergency?

7. If you can run 100 meters in 10 seconds, how long, in days, hours, and minutes, does it take you to run 12,800,000 meters?

8. If Lucy runs 3 mph and Jan runs 7 mph, when will Jan catch up with Lucy if she gives Lucy a 10 minute head start?

9. Steven ran a 12-mile race at an average speed of 8 miles per hour. If Adam ran the same race at an average speed of 6 miles per hour, how many minutes longer than Steve did Adam take to complete the race?

10. A 555-mile, 5-hour plane trip was flown at two speeds. For the first part of the trip, the average speed was 105 mph. Then the tailwind picked up, and the remainder of the trip was flown at an average speed of 115 mph. For how long did the plane fly at each speed?

11. An executive drove from home at an average speed of 30 mph to an airport where a helicopter was waiting. The executive boarded the helicopter and flew to the corporate offices at an average speed of 60 mph. The entire distance was 150 miles; the entire trip took three hours. Find the distance from the airport to the corporate offices.

12. A car and a bus set out a 2 p.m. from the same point, headed in the same direction. The average speed of the car is 30 mph slower than twice the speed of the bus. In two hours, the car is 20 miles ahead of the bus. Find the rate of the car.

13. A passenger train leaves the train depot 2 hours after a freight train leaves the same depot. The freight train is traveling 20 mph slower than the passenger train. Find the rate of each train, if the passenger train overtakes the freight train in three hours.

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