"What did the vampire doctor say to his patient?"

Worksheet - Solving log equations using properties

Recall: Properties of Logs
1. \( \log_b MN = \log_b M + \log_b N \)
2. \( \log_b \frac{M}{N} = \log_b M - \log_b N \)
3. \( \log_b M^x = x \log_b M \)

To solve a log equation, change the log to an exponential using the Circle Rule.

n: \( \log_2(2x-1) = 3 \)  \quad \text{Circle Rule}

\[
3^3 = 2x - 1 \\
27 = 2x - 1 \\
+1 = +1 \\
28 = 2x \\
14 = x
\]

0: \( \log_4 (3x-11) = \log_4 (x-3) \)

since the equation is \( = \), it would be safe to assume that:

\[
\begin{align*}
3x-11 &= x-3 \\
-x &= -x \\
2x-11 &= -3 \\
+11 &= +11 \\
2x &= 8 \\
\therefore x &= 4
\end{align*}
\]

H: \( \log_6 x + \log_6 3 = 2 \)  \quad \text{Prod Property}

\[
\log_6 3x = 2 \\
6^2 = 3x \\
36 = 3x \\
12 = x
\]

S: \( \log_2 (x-3)^6 = 6 \)  \quad \text{Pwr Prop}

\[
\begin{align*}
3 \log_2 (x-3) &= 6 \\
\frac{3}{3} &= \frac{6}{3} \\
\log_2 (x-3) &= 2 \\
2^2 &= x-3 \\
4 &= x-3 \\
+3 &= +3 \\
\therefore x &= 1
\end{align*}
\]

4: \( \log_3 (x-1) = \log_3 (x+1) \)  \quad \text{Circle Rule}

\[
\begin{align*}
x-1 &= x+1 \\
-x &= +x \\
-2x &= 2 \\
\therefore x &= -1
\end{align*}
\]
F: \(4 \log_8 x = 2 \log_8 9\)  
\[\log_8 x^4 = \log_8 81\]  
\[\sqrt[4]{x^4} = 81\]  
\[x = \pm 3\]  
However \(x = 3\) (\(\log -3 \Rightarrow \text{undefined}\))

I: \(\log x + \log (x+2) = \log 3\)  
Product Prop
\(\log x(x+2) = \log 3\)
\(\log(x^2+2x) = \log 3\)
\(\log(x^2+2x) - \log 3 = 0\)
\(\log\left(\frac{x^2+2x}{3}\right) = 0\)
Circle Rule \(\Rightarrow\) understood to be base 10

\(10^0 = \frac{x^2+2x}{3}\)

\(3\left(1 = \frac{x^2+2x}{3}\right)\)
Multiply by 3 to get rid of fraction

\[-3 = x^2+2x\]
Set = 0

\[-3 = x^2+2x\]
Factor

\(0 = x^2+2x-3\)
\(0 = (x+3)(x-1)\)
\(x+3=0\), \(x-1=0\)
\(x = -3\), \(x = 1\)

Can't take \(\log\) of a neg #
\(\log -3 \Rightarrow\) Not possible

A: \(2 \log_2 (x+6) - \log_2 16 = 2\)  
Pur Prop
\(\log_2 (x+6)^2 - \log_2 16 = 2\)
Quot Prop
\(\log_2 \left(\frac{(x+6)^2}{16}\right) = 2\)
Circle Rule

\(2^2 = \frac{(x+6)^2}{16}\)

\(16 \left(\log 4 = \frac{(x+6)^2}{16}\right)\)
\(\log\) both sides

\(\pm 8 = x + 6\)
\(-6\)
\(-6 + x \Rightarrow x = -6 + 8 = 2\)
\(x = -6 - 8 = -14\) Not Possible
\(2\log_2 (-14 + 6) \Rightarrow\) undefined
I dropped the log in the next step because since the log on the left was equal to the log on the right it was same to assume that what I was taking the log of on each side was equal as well.

\[
\text{C: } \log_4 \left( \frac{x^2 - 4}{x+2} \right) = \log_4 (x-2) = 2 \quad \text{Quot Prop}
\]

\[
\log_4 \left( \frac{4x}{x+2} \right) = 2 \quad \text{Factor & Simplify}
\]

\[
\log_4 (x-2) = 2 \quad \text{Circle Rule}
\]

\[
4^2 = x - 2
\]

\[
16 = x - 2 + a + a
\]

\[
18 = x
\]

\[
\text{T: } \log_2 \left( \frac{5x+7}{x} \right) = \log_2 x = 2 \quad \text{Quot Prop}
\]

\[
\log_2 \left( \frac{5x+7}{x} \right) = 2 \quad \text{Circle Rule}
\]

\[
a^2 = \frac{5x+7}{x}
\]

\[
x \left( \frac{4 = 5x+7}{x} \right)
\]

\[
4x = 5x + 7
\]

\[
-5x - 5x
\]

\[
-x = 7
\]

\[
x = \frac{-7}{-5} = \frac{7}{5} \quad \text{Not Possible } \log_2 x = \text{undefined} \quad \text{No Solution}
\]

\[
\text{P: } \log(x+5) = \log(x-1) = \log(x+2) = \log(x-3) \quad \text{Quot Prop}
\]

\[
\log \frac{x+5}{x-1} = \log \frac{x+2}{x-3}
\]

\[
\frac{x+5}{x-1} = \frac{x+2}{x-3}
\]

\[
\text{FOIL}
\]

\[
(x+5)(x-3) = (x+2)(x-1)
\]

\[
x^2 - 3x + 5x - 15 = x^2 + 2x - x - 2
\]

\[
x^2 + 8x - 15 = x^2 + x - 3
\]

\[
-x^2 - x + 2 - x^2 + x + 2
\]

\[
x - 13 = 0
\]

\[
x = 13
\]